

Two dimensional anisotropic non Fermi-liquid phase of coupled Luttinger liquids

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We show using bosonization techniques, that strong forward scattering interactions between one dimensional spinless Luttinger liquids (LL) can stabilize a phase where charge-density wave, superconducting and transverse single particle hopping perturbations are irrelevant. This new phase retains its LL like properties in the directions of the chains, but with relations between exponents modified by the transverse interactions, whereas, it is a perfect insulator in the transverse direction. The mechanism that stabilizes this phase are strong transverse charge density wave fluctuations at incommensurate wavevector, which frustrates crystal formation by preventing lock-in of the in-chain density waves.

Interacting fermions in one dimension can exhibit Luttinger liquid behaviour where, in contrast to Fermi liquids, all the low lying excitations are collective modes [1]. The search for such non-Fermi liquid behaviour in higher dimensions prompted several authors to study the problem of coupled Luttinger liquids. But renormalization group (RG) studies, such as [2] found the coupling to destabilize the Luttinger liquid behaviour. It was however argued by Anderson *et al.* that, in spite of its relevance in the RG sense, electron transverse hopping may be incoherent [3], allowing for a non-Fermi liquid in dimensions greater than one. In addition to these theoretical motivations, interacting Luttinger liquids could well arise in experimental systems like quasi one dimensional organic conductors and in “ropes” of nanotubes. Recently, coupled one dimensional systems have also emerged in the stripe phases of Quantum Hall systems in higher Landau levels [4], and in the cuprates [5].

In this paper, we will revisit the problem of coupled Luttinger liquids, and address the question of the existence of a stable two dimensional phase that retains some of the properties of the one dimensional Luttinger liquid. We consider this issue in detail within the RG framework using bosonization, for the case of spinless fermions. Although most of the physical systems of interest are of spinful (rather than spin polarized or spinless) fermions, that case is technically more involved due to the presence of exchange interactions between the chains, and we postpone its study to [6]. The case of an infinite set of coupled spinless Luttinger liquids is simpler, but should still capture some of the general physics present in the spinful case. There is also a possibility that such systems may be directly realized by spin polarizing a layered quasi-one-dimensional system of low electron density with an in plane magnetic field; or in the spin polarized Luttinger liquid formed by Zeeman split quasiparticles bound to the superconductor vortex core [7,8]. The new ingredient in our study is the inclusion of strong forward scattering interactions between the chains. We then find that an anisotropic phase, which is Luttinger liquid like along the chains but a perfect charge insulator in the transverse di-

rection, is stable against a range of instabilities including those that could lead to a superconductor, crystal or 2D fermi liquid phase. Hence this new phase is a non-Fermi liquid phase in two dimensions that is highly anisotropic. In the direction of the chains the correlation functions have power law forms with nontrivial exponents, as in a Luttinger liquid. However, the relations between exponents are modified, as compared with completely decoupled Luttinger liquids, by the strong forward scattering interactions between the chains. We will call this phase the *sliding Luttinger liquid (SLL)*. It is analogous to the sliding phase found by O’Hern *et al.* [9] in the related problem of XY models coupled by suitable gradient interactions, which motivated our approach. In contrast to [10], we consider transverse hopping operators when establishing the stability of the SLL.

The stable sliding Luttinger liquid fixed points that we find occur close to an instability towards transverse charge density wave ordering. The instability occurs when the stiffness of the density fluctuation mode at a particular transverse wave-vector vanishes. We propose a physical mechanism that is responsible for stabilizing this phase - i.e. that the proximity to the transverse charge density wave state induces strong fluctuations of the local in-chain density, which frustrates crystal formation. This mechanism is most effective if the wavelength of the transverse charge density wave phase is incommensurate with the spacing between the chains.

The existence of this phase may at first sight contradict previous results on coupled Luttinger liquids. However, previous approaches focused either on the simpler case of two interacting chains [2], or on the perturbative effect of these transverse interactions. Indeed, as we will show this phase can exist when strong forward scattering couplings between nearest and (at least) second nearest neighbour chains are taken into account.

The sliding Luttinger liquid. We consider an anisotropic 2D system composed of parallel chains (labelled by an integer i) with spinless Luttinger liquids in each chain. The bosonized form of the fermion operators

near the right and left fermi points are:

$$\psi_{R/L,i}(x) = \frac{1}{\sqrt{4\pi\epsilon}} e^{i\phi_{R/L,i}(x)} \chi_{R/L,i}$$

where ϵ is some intra-chain cut-off, the $\chi_{R/L,i}$ are the Klein factors and x is the coordinate along the chain. The Luttinger liquid on each chain is described by the usual lagrangian:

$$\mathcal{L}_0^{(i)} = \frac{K_0}{2} [(\partial_t \Theta_i(x, t))^2 - (\partial_x \Theta_i(x, t))^2] \quad (1)$$

where $\Theta_i = (\phi_{L,i} - \phi_{R,i})/\sqrt{4\pi}$, and K_0 is an interaction dependent constant with $K_0 > 1$ for repulsive intra-chain interactions. The sound velocity on each chain has been set to unity. In the following, we will also use the dual field $\Phi_i(x, t) = (\phi_{L,i} + \phi_{R,i})/\sqrt{4\pi}$.

We now add forward scattering interactions between the chains which correspond to couplings between the long wavelength components of the densities $\rho_i(x, t)$, and of the currents $J_i(x, t)$:

$$\mathcal{L}_{int} = 2\pi \sum_{i \neq j} [J_i \bar{K}_{ij}^J J_j - \rho_i \bar{K}_{ij}^\rho \rho_j] \quad (2)$$

where the interactions are assumed local in x and time. Using the bosonization relations $\sqrt{4\pi} \rho_i(x, t) = \partial_x \Theta_i(x, t)$ and $\sqrt{4\pi} J_i(x, t) = -\partial_t \Theta_i(x, t)$, we obtain the bosonized form of the lagrangian $\mathcal{L}_{tot} = \sum_i \mathcal{L}_0^{(i)} + \mathcal{L}_{int}$ for the interacting Luttinger liquids :

$$\mathcal{L}_{tot} = \frac{1}{2} \sum_{i,j} [(\partial_t \Theta_i) K_{ij}^J (\partial_t \Theta_j) - (\partial_x \Theta_i) K_{ij}^\rho (\partial_x \Theta_j)] \quad (3)$$

where the coupling matrices are defined by $K_{ij}^J = K_0 \delta_{ij} + \bar{K}_{ij}^J$, $K_{ij}^\rho = K_0 \delta_{ij} + \bar{K}_{ij}^\rho$. By introducing Fourier transforms in the direction transverse to the chains, this lagrangian can be rewritten as

$$\mathcal{L}_{tot} = \int_{q_\perp} \frac{K(q_\perp)}{2} \left[\frac{1}{v(q_\perp)} |\partial_t \Theta_{q_\perp}|^2 - v(q_\perp) |\partial_x \Theta_{q_\perp}|^2 \right] \quad (4)$$

with the notation $\int_{q_\perp} = \int_{-\pi}^{\pi} \frac{dq_\perp}{2\pi}$ (the transverse chain spacing has been set to unity). The stiffness $K(q_\perp)$ is defined by $K(q_\perp) = \sqrt{K^J(q_\perp) K^\rho(q_\perp)}$ and the velocity $v(q_\perp) = \sqrt{K^\rho(q_\perp) / K^J(q_\perp)}$. Note that Lorentz invariance corresponds to $K^\rho(q_\perp) \propto K^J(q_\perp)$ (like in the isotropic model studied in [9]). In that case, all modes have the same velocity.

The lagrangian (3) is invariant under the transformations $\Phi_i \rightarrow \Phi_i + c_i$ and $\Theta_i \rightarrow \Theta_i + d_i$ where c_i and d_i are constants on each chain. We will thus call the corresponding fixed point a *sliding Luttinger liquid (SLL)*

fixed point [11]. From this symmetry we deduce that the total numbers of left (right) moving fermions on each chain are good quantum numbers and expectation values of operators that change these - such as $\langle \psi_{L,i}^\dagger \psi_{L,j} \rangle$ for $i \neq j$ - are necessarily zero in this phase. This corresponds to a perfect charge insulator in the transverse direction. Density (and current) correlations in the transverse direction are however nontrivial. For short ranged density and current interactions, they decay exponentially with separation between the chains. The low energy modes are density oscillations (sound) with dispersion $E(q_\parallel, q_\perp) = v(q_\perp) |q_\parallel|$ (where q_\parallel is the wavevector along the chain) which in general will propagate both parallel and perpendicular to the chains. These modes can, for instance, transport heat perpendicular to the chains although the system is a perfect charge insulator in that direction. Correlation functions along the chains exhibit power law behaviour as in the Luttinger liquid. However the exponents now depend on the function $K(q_\perp)$ rather than on a single number as in the case of completely decoupled Luttinger liquids (1). Therefore, relations between exponents that are valid for decoupled Luttinger liquids no longer hold in this case.

As this phase is described by a gaussian lagrangian, we can study perturbatively the possible relevance of various operators to ascertain its stability.

Transverse hopping operators. In the usual case of two chains, it is easily shown that the most relevant operators correspond either to single particle (SP), particle-hole (CDW) or pair hopping (SC) [2]. It is thus natural to first focus on these operators in our stability analysis. They are defined respectively by

$$\delta \mathcal{L}_{SP} = \sum_{i,j} t_\perp^{ij} \left(\psi_{R,i}^\dagger \psi_{R,j} + \psi_{L,i}^\dagger \psi_{L,j} + h.c. \right) \quad (5a)$$

$$\delta \mathcal{L}_{CDW} = \sum_{ij} g_{CDW}^{ij} \left(\psi_{R,i}^\dagger \psi_{L,i} \psi_{L,j}^\dagger \psi_{R,j} + h.c. \right) \quad (5b)$$

$$\delta \mathcal{L}_{SC} = \sum_{ij} g_{SC}^{ij} \left(\psi_{L,i}^\dagger \psi_{R,i}^\dagger \psi_{R,j} \psi_{L,j} + h.c. \right) \quad (5c)$$

Upon bosonization, the corresponding operators read:

$$\mathcal{O}_{SP}^{ij} = \cos \sqrt{\pi} (\Phi_i - \Phi_j) \cos \sqrt{\pi} (\Theta_i - \Theta_j) \quad (6a)$$

$$\mathcal{O}_{CDW}^{ij} = \cos \sqrt{4\pi} (\Theta_i - \Theta_j) \quad (6b)$$

$$\mathcal{O}_{SC}^{ij} = \cos \sqrt{4\pi} (\Phi_i - \Phi_j) \quad (6c)$$

The dimensions of these operators at the SLL fixed point (4) are readily evaluated and are given by :

$$\eta_{CDW}^{(N)} = 2 \int_{q_\perp} \frac{1 - \cos(Nq_\perp)}{K(q_\perp)} \quad (7a)$$

$$\eta_{SC}^{(N)} = 2 \int_{q_\perp} (1 - \cos(Nq_\perp)) K(q_\perp) \quad (7b)$$

$$\eta_{SP}^{(N)} = \frac{1}{4} (\eta_{CDW} + \eta_{SC}) \quad (7c)$$

where $N = |i - j|$. A necessary condition for this sliding LL phase to be stable is thus that

$$\eta_{\text{CDW}}^{(N)}, \eta_{\text{SC}}^{(N)} > 2 \quad \text{and} \quad \eta_{\text{CDW}}^{(N)} + \eta_{\text{SC}}^{(N)} > 8 \quad (8)$$

for all $N \geq 1$.

Stability of the SLL phase. Before discussing the stability of SLL fixed points, let us first discuss the domain of validity of our RG approach. We are considering the scaling dimension of perturbing operators around the fixed points defined by the action (4). This action is defined provided the stiffness $K(q_\perp)$ is positive everywhere in $[-\pi, \pi]$. Our study is thus restricted to the corresponding subspace of $K(q_\perp)$. At the boundary of this subspace $K(q_\perp)$ vanishes for some $q_\perp^0 \in [0, \pi]$ [12]. The density correlations of transverse wavevector q_\perp^0 then diverge, signalling an instability towards transverse charge density wave ordering. In the transverse CDW, $\langle \rho_{q_\perp} \rangle \neq 0$, so then the total charge density on each chain is a function of the transverse position i [13]. That the boundary corresponding to this transition will play a crucial role in the following analysis can be seen by inspecting the dimensions (7) : $\eta_{\text{CDW}}^{(N)}$ can be significantly increased by the presence of the pole in the integrand (root of $K(q_\perp)$) with $\eta_{\text{SC}}^{(N)}$ being not much affected [9]. Hence it may be possible to have these operators irrelevant (i.e. with dimension greater than 2), for parameters in the vicinity of this boundary.

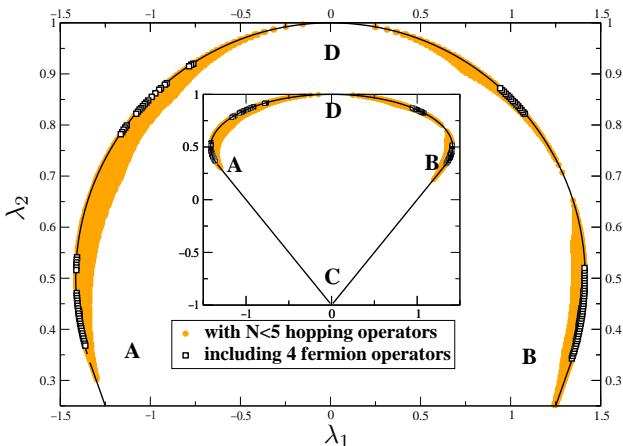


FIG. 1. SLL fixed points for the model in (9). The allowed parameter range for which $K(q_\perp)$ is positive is within the curve ACBD. For each point shown, there exists a finite range of K_0 for which the SLL is found to be stable against (a) CDW, SC and SP perturbing operators of eq.(5) with $N \leq 4$ shown by the small dots and (b) including general four point operators with $N \leq 10$ shown as squares.

Stability analysis of a model $K(q_\perp)$ Here we shall consider the stability analysis for a concrete model of $K(q_\perp)$. We thus look for a natural restriction to a finite number of terms of the Fourier expansion, that may allow a stable

SLL fixed point. The simplest case turns out to be

$$K(q_\perp) = K_0[1 + \lambda_1 \cos(q_\perp) + \lambda_2 \cos(2q_\perp)] \quad (9)$$

(As we will discuss later, the model with just λ_1 included does not possess a stable SLL phase). The requirement of positivity of $K(q_\perp)$ restricts the range of λ_1 and λ_2 , as shown in the Figure 1, to the region within the boundary ADBC (henceforth simply denoted by \mathcal{B}). The stability of the SLL to the perturbations (6) with $1 \leq (N = |i - j|) \leq 4$ is first determined [14]. The exponents in eq. (7) are numerically evaluated and the results are shown in Figure 1 which is a two dimensional representation of the $(K_0, \lambda_1, \lambda_2)$ space. The points marked are those for which there exists a range of K_0 values where the above operators are all irrelevant. The corresponding values of K_0 are all greater than one (repulsive interactions on the chains). All the stable fixed points are thus found close to the boundary \mathcal{B} .

In an attempt to define stable RG fixed points, one may worry about more general perturbing operators. Indeed, inclusion of the operators (6) with $5 \leq N \leq 10$ does not significantly change the results. However, it turns out that general four point operators such as, for example, simultaneous hopping of fermions from chain i to $i + 1$ and from j to $j + 1$ [15], has a dramatic effect on the stability region which is now much reduced. Thus, these operators are found to be relevant at a large number of the previously stable points. There are however some remaining fixed points, clustered close to the boundary \mathcal{B} , which are shown as squares in Figure 1.

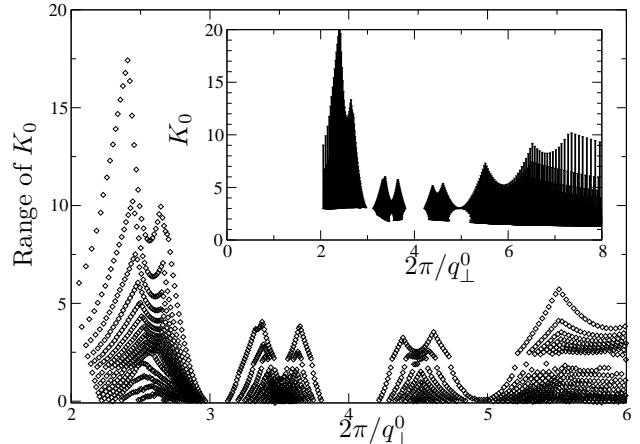


FIG. 2. Range of K_0 corresponding to stable SLL fixed points as a function of the transverse CDW wavelength $2\pi/q_\perp^0$. Inset shows actual values of K_0

Thus, as expected from the argument given above, all stable fixed points are found close to the boundary \mathcal{B} . This can be physically understood as follows.

Let us recall that anywhere on \mathcal{B} the system is on the verge of a transverse CDW instability. Such a transverse CDW would frustrate crystallization of the fermions since $(2k_F)^{-1}$, which controls the spacing of the particles on

each chain, is now a function of the chain index i . Strong fluctuations of this kind prevent the locking in of density modulations along the chain, hence defeating the crystal instability and stabilizing the SLL. This mechanism will be less effective if the wavevector q_\perp^0 is commensurate with the transverse lattice *i.e.* if $q_\perp^0 = 2\pi m/n$ for some integers m,n . Then, the longitudinal density waves on chains separated by n can lock in and give rise to the crystal instability (provided n is not too large). This idea is confirmed on inspecting the range of K_0 for which the SLL phases exist. In Figure 2 the corresponding range of K_0 is plotted as a function of the transverse CDW instability period $2\pi/q_\perp^0$ (that it is nearest to). Drastic reduction of the SLL stability occurs at commensurate transverse wavevectors. This also allows us to understand the absence of stable SLL in the model with $\lambda_2 = 0$. Then, the only existing transverse CDW has wavevector π (for $\lambda_1 = 1$) and hence for strong enough repulsive interactions, we expect the longitudinal density waves on next nearest neighbor chains ($N=2$) to lock and lead to a crystalline instability. This turns out to be a correct expectation as, by including the $N=2$ CDW operator, we do not find a stable SLL in this model.

The physical mechanism identified here allows us to generalize our results to models beyond the simple ones considered so far, including Luttinger liquids coupled in three dimensions. In general we expect the SLL to be stabilized for renormalized couplings in the vicinity of an incommensurate transverse CDW transition.

An approach very similar to the one taken in this paper can be applied to bosonic models at any filling, coupled anisotropic spin chains, or vortex lattices in anisotropic superconductors. These, together with the more complicated case of spinful fermions will be discussed in a forthcoming publication [6]. As disorder is known to modify significantly the behaviour of a single Luttinger liquid, it may also be highly interesting to study its effect on the new phase described in this paper.

In conclusion we have studied in detail the occurrence of the sliding Luttinger liquid phase in a simple three parameter model. We find that the region of stability of this phase is restricted to be close to the boundary where the harmonic boson theory breaks down and a soft mode appears, that signals a transverse CDW instability. Strong incommensurate transverse CDWs are particularly effective at frustrating crystal formation and enhancing the stability of the SLL. We expect these conclusions to apply also to more general models than the simple one considered in this paper.

During the completion of this work, we became aware of a similar, but nevertheless different, study by Emery et al. [16].

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- [12] The case of double roots is considered in [6].
- [13] This is analogous to the spin 1/2 XXZ chain in the bosonized representation, where the boson stiffness K goes to zero at the isotropic ferromagnet point, signalling the transition to an ordered ferromagnetic state. See I. Affleck in *Fields, Strings and Critical Phenomena*, Les Houches Summer School 1988, E. Brezin and J. Zinn-Justin Eds. (North Holland, New York 1990).
- [14] Choosing the set of perturbing operators for a physical situation is an involved issue, which will depend on details of the system and the small but nevertheless finite temperature at which it is considered. Then, operators that have small bare values and are not further generated in the RG could be unimportant, even if relevant.
- [15] More precisely, we consider operators of the form $\psi_{\alpha i}^\dagger \psi_{\beta j}^\dagger \psi_{\gamma k} \psi_{\delta l}$ where $\alpha = \pm 1$ for (L/R) and $\alpha + \beta = \gamma + \delta$ by momentum conservation. We have taken the maximum separation between any two fermion operators to be ≤ 10 . The ones that destabilize the SLL points usually involve simultaneous hops on pairs of nearest neighbour chains. These could well be present or generated in physical systems.
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